TRINITY COLLEGE

Mathematics Specialist Units 3 & 4
Test 4 2016

Section 1 Calculator Free

Vector Calculus and
Integration using Trigonometric Identities and Substitution

STUDENT'S NAME: _____

DATE: Friday 20th May **TIME:** 25 minutes **MARKS**: 27

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet.

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

Evaluate the following:

(a) $\int_{-\pi}^{\pi} \cos x \, dx$ [2]

(b) $\int_{-\pi}^{\pi} |\cos x| \, dx$ [3]

2. (5 marks)

Evaluate the definite integral:

$$\int_{-1}^{\frac{1}{3}} \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx, \qquad \text{given} \quad \int f'(x)(f(x))^n dx = \frac{(f(x))^{n+1}}{n+1} + c$$

3. (9 marks)

Determine the following integrals:

(a) $\int \frac{\sin(2x)}{\cos x}\,dx$ [4]

(b) $\int \sin^4(2x)\,dx$ [5]

4. (8 marks)

Determine the following using the given substitution:

(a) $\int \frac{dx}{1+x^2}$ Let $x = \tan\theta$ [4]

(b) $\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$ Let $x = 2\sin\theta$ [4]

End of Questions

TRINITY COLLEGE

Mathematics Specialist Units 3 & 4
Test 4 2016

Section 2 Calculator Assumed

Vector Calculus and
Integration using Trigonometric Identities and Substitution

STUDENT'S NAME: _____

DATE: Friday 20th May **TIME:** 25 minutes **MARKS**: 27

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet retained from Section 1.

Special Items: Drawing instruments, templates, three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment).

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (4 marks)

You will recall that in Question 2 you were asked to evaluate the definite integral:

$$\int_{-1}^{\frac{1}{3}} \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$$

You may well have obtained the result of $-\frac{16}{3}$.

Now use your calculator to evaluate the above definite integral. Comment on your result.

6. (13 marks)

The orbit of a planet around its sun is given by the position vector

$$\mathbf{r}(t) = \cos\left(\frac{\pi t}{200}\right)\mathbf{i} - 2\sin\left(\frac{\pi t}{200}\right)\mathbf{j}$$

where *t* is time measured in Earth days and distance is in appropriate astronomical units.

(a) Determine $\mathbf{r}(0)$ and $\mathbf{r}(400)$. Hence calculate the length of the planet's year. [3]

(b) Show that the distance of the planet from its sun is $d = \sqrt{1 + 3\sin^2\left(\frac{\pi t}{200}\right)}$ [2]

(c) At what time during the planet's year, is it a maximum distance from its sun? [2]

(d) Determine the planet's orbiting speed when it is at its maximum distance from its sun.

[2]

(e) Show that the acceleration vector is a scalar multiple of the position vector. [2]

(f) State the Cartesian equation of the path of the planet. [2]

7. (10 marks)

An object is launched from a point with position vector $\mathbf{r}(0) = 18\mathbf{i} + 4\mathbf{j}$ metres. The velocity vector of the object, *t* seconds after projection, is given by $\mathbf{v}(t) = -\mathbf{i} - \frac{1}{2\sqrt{16-t}}\mathbf{j}$ ms^{-1} .

(a) Determine the position vector of the object at time *t* seconds. [2]

(b) Determine the position vector of the point where the object hits the ground. [3]

(c) Determine the speed and the direction of the object at $t = 12$ seconds. [3]

(d) Calculate the total distance travelled by the object in the first 12 seconds. [2]

End of Questions